Topics in Algorithms 2007

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Tree Embeddings

Projections

- Can vertices in a given weighted graph G be mapped to vertices in any prob distribution over edge-weighted trees H so that all distances (i.e., shortest paths) only increase, but not by too much?
- Distortion: max stretch over all edges in G

Upper Bound

- Fakcharoenphol, Rao, Talwar upper bound of log n for probabilistic embeddings

Upper Bound

- Hierachical cluster decomposition
  - Parameter r (radius) which shrinks with time
  - Each cluster C at time i splits into further clusters at time i+1 as follows
  - Every vertex in C attaches to a center within distance r
  - Each center gives a new sub cluster
  - The radius of each new sub cluster is at most r

Upper Bound

- The cluster decomposition tree
  - Root: cluster with all vertices
  - Internal node: a cluster C
  - Children: subclusters of C
  - Weight of edge from C to its children: $2r$ (diameter of C) , guarantees expansion
  - Leaf: individual nodes
  - What is the expected Distortion (over random choices)??

Bounding Expected Distortion

If vertices u and v get split when r=x, and r shrinks geometrically then distortion is O(x)

Danger: x>>d(uv)
Need to make prob of splitting uv small when r>>d(uv)

Randomness

- We need randomness to defeat the lower bound. Where does this randomness come from?
- How are centers chosen?
- Each vertex picks an arbitrary/random center within distance r? Doesn't work because vertices close by can split very early on
- Flip around and consider centers one by one in random order, take all vertices within r of a center to create a new cluster
- Now if uv are close to each other than hopefully they will both attach to the same center and not split early

Randomness

- What happens if there is only one center w within r of u and and that v is just outside the reach of w;
- Then irrespective on randomness in center choice, uv will split early
- So we need more randomness; introduce randomness in r
- r should have a distribution spread over a range that is proportional to its mean; for instance take r0 uniformly in [1,2] at the very end and r is $r0*2^i$ where i runs back in time; so at time i, r is uniform in $2^i..2^{i+1}$
- So if d(uv) is much smaller than r~d(wu),d(wv) then this randomness ensures that the chances of uv being split is down to d(uv)/r

Bounding Expected Distortion

2 parameters

- Which vertex splits uv (if 2 vertices take the closer one)?
- At what r does the split happen?

$\Sigma_{\{x,w\}}$ x*Prob(split happens due to w when r=x..2x)

- $\Sigma_{\{x,w\}}$ x*0 if d(w,u),d(w,v) are both >r or both <r
- <=$\Sigma_{\{x,w\}}$ x*1/i if d(w,u)>r and d(w,v)<r and w is the ith closest vertex to edge uv
- =$\Sigma_{\{x,w\}}$ x*1/i* d(uv)/x
- log n * d(uv) !!