Fast Algorithms for Connectivity Problems in Networks

Steiner Cuts, Gomory-Hu Trees, and Edge Splitting

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Strand Life Sciences

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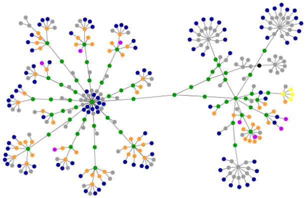
The Setting

- A network G of n nodes and m edges.
- Many nodes, relatively fewer connections.
- Unweighted edges.
- Redundancy requirement (k) much smaller than n.
- Reduce algorithms with time complexity $O(n^2)$ or more to $\tilde{O}(n \text{ poly}(k))$



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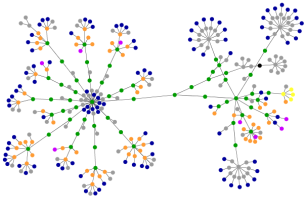

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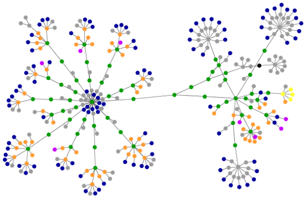

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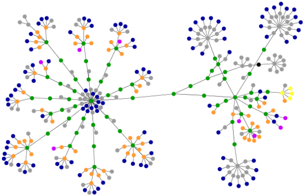

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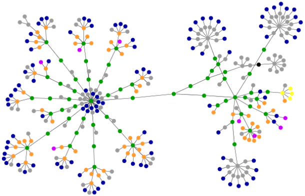
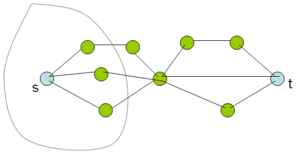


$s - t$ Edge Connectivity/Min-Cut

- The maximum number of edge disjoint paths between two vertices.
- Equals the min-cut separating the 2 vertices
- Can be computed in $O(mk)$ time via Ford-Fulkerson max flows, k is the min-cut size



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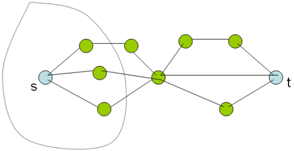

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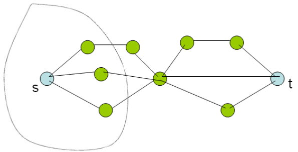

Global Edge Connectivity/Min-Cut

- The minimum edge connectivity over all vertex pairs.
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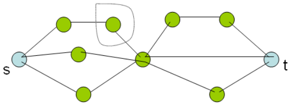
- Best Deterministic Algorithm: $O(mk)$ by Gabow where k is the global min-cut size; near linear Monte Carlo algorithm by Karger.



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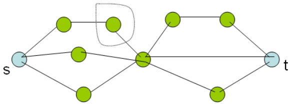

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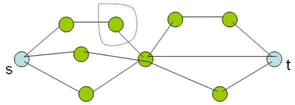

Steiner Edge Connectivity/Min-Cut

- S is a subset of interesting Steiner vertices.
- The minimum edge connectivity over all vertex pairs from S.
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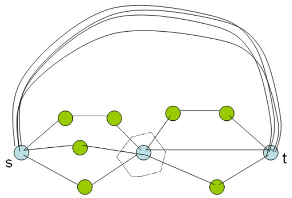
- Question: Is an $O(mk)$ algorithm possible, where k is the Steiner min-cut?



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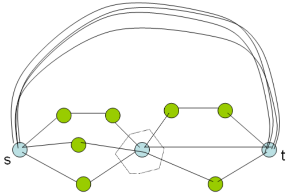

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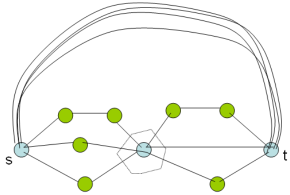

The Gomory-Hu Tree

- A tree which carries min-cut information for all pairs of vertices.
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- Time taken is $O(n^3)$ or more.
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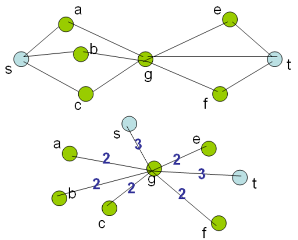

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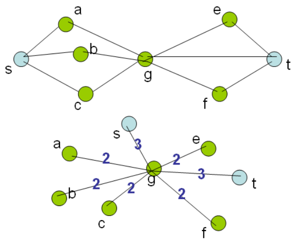

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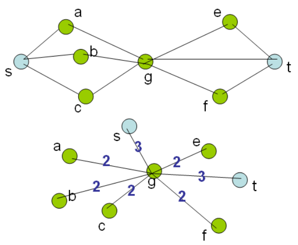

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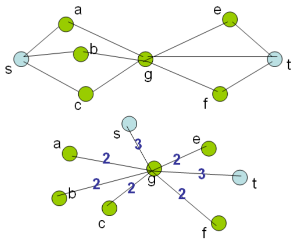

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- Create two recursive sub-problems
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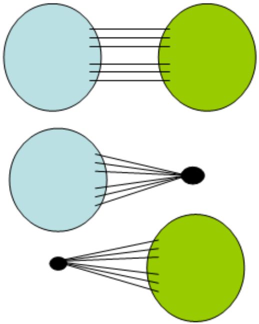

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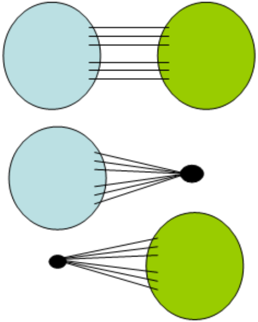

Edge Splitting

- Pair edges incident on a vertex and remove it
- Ensure that pairing conserves the global min-cut of the remaining vertices
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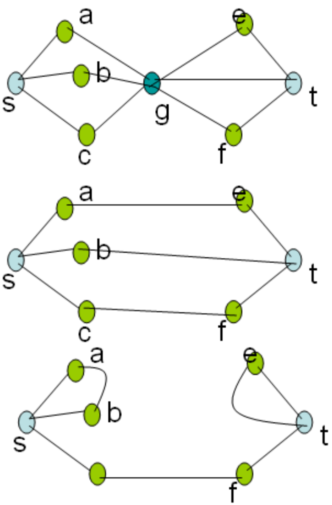
- Lovasz/Mader: Yes, for eulerian directed graphs and for even degree vertices in undirected graphs



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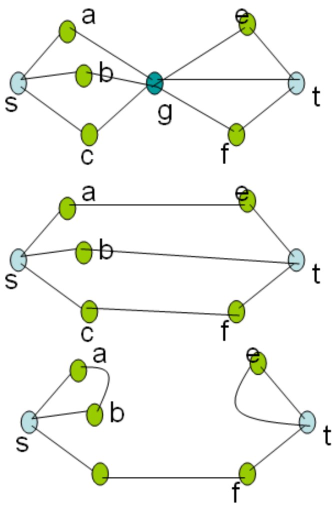

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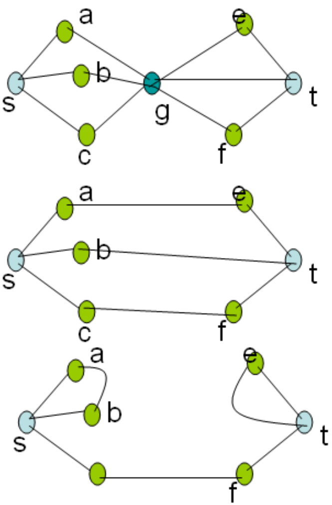

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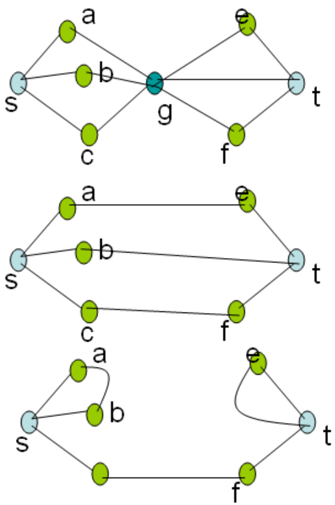

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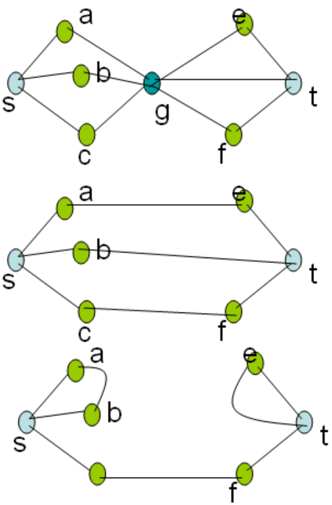

Edge Splitting and Steiner Min-Cuts

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Edge Splitting Complexity

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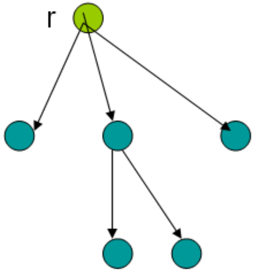
Edmonds' Arborescences

- Direct an undirected graph by orienting each edge in both directions.
- Arborescence: A spanning tree with edges directed away from an arbitrary root
- Build as many edge-disjoint arborescences as possible rooted at some chosen vertex r
- $\tilde{O}(n^2)$ time construction by Gabow.



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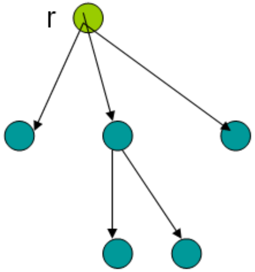

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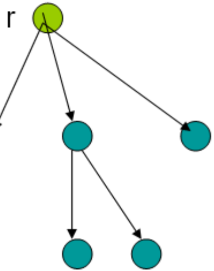
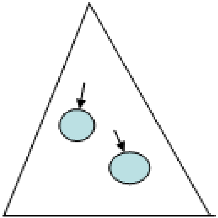


Global Min-Cut Characterizations from Edmonds' Arborescences

- Any subset of vertices not containing the root must have in-degree at least 1 in each tree.
- Existence of k edge-disjoint arborescences implies global min-cut $\geq k$



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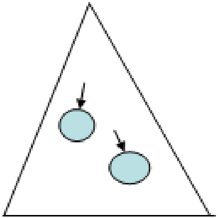

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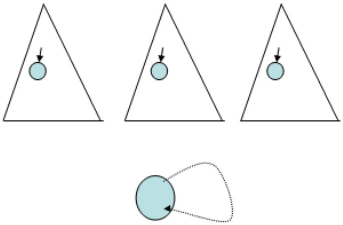

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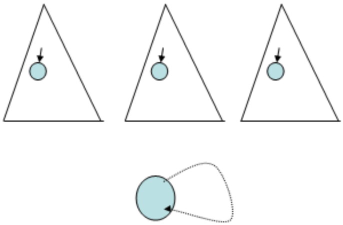

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- Swaps need to be constrained to maintain the arborescence property.
- Relax the property that edges are directed away from the root. Instead insist that in-degree of a vertex over all directionless trees equals k
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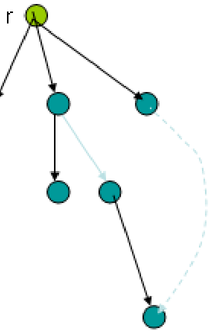

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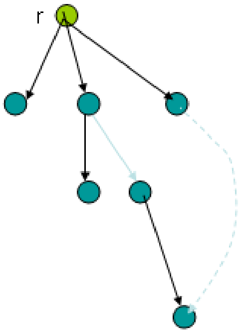

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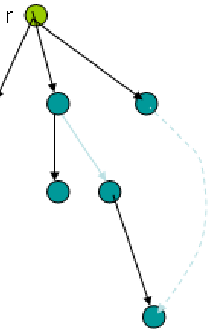

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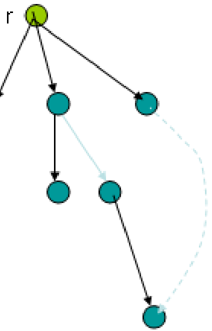

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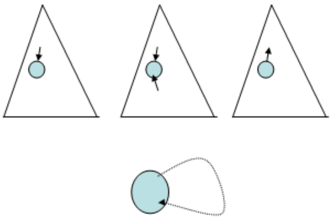

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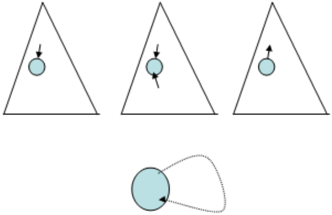

Edmonds' Directionless Trees Construction Algorithm

- Suppose i trees have been constructed and the $i+1$th tree is a forest.
- For each component in the $i+1$th tree, run a closure algorithm to identify a subset S of vertices not containing the root, contiguous in all trees, and with all unused edges directed into S having both endpoints in S.
- If no such subset can be found then a sequence of swaps results in components in the $i+1$th tree connecting up.
- $\tilde{O}(m)$ time for the $i+1$th tree so $\tilde{O}(mk)$ overall; and m can be made $O(nk)$ via Nagamochi-Ibaraki

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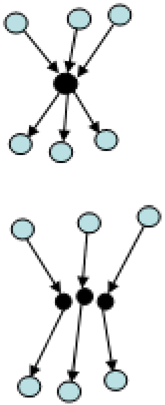
- Split-off non-Steiner (black) vertices via arbitrary pairing of edges (the global min-cut of the remaining white Steiner vertices could drop in the process)
- Each edge is actually a path with internal blacks
- Run Gabow's construction of Edmonds' directionless trees on this new graph comprising only whites (ignore blacks)



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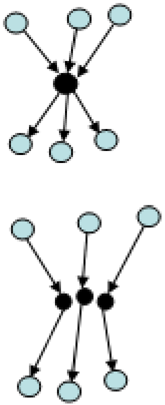

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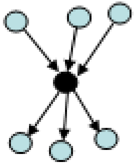

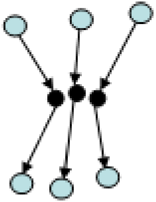

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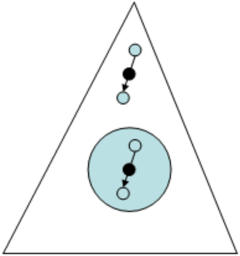

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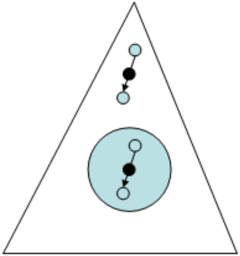

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- We introduce a mating operation (i.e., revision of pairing) whenever contiguity on blacks is violated
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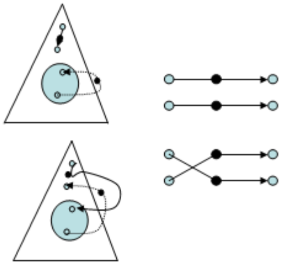

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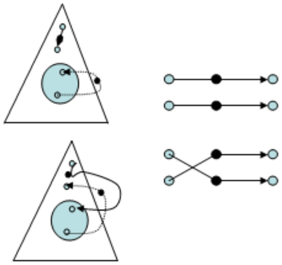

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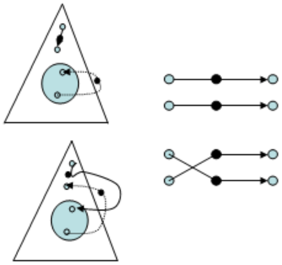

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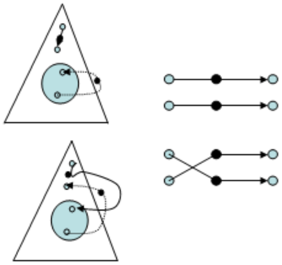

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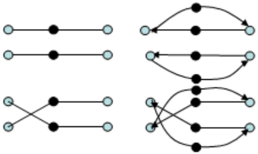

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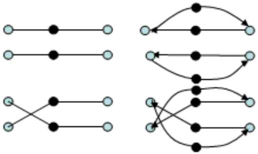

Faster Edmonds' Arborescence Construction

- Goal: Construct k arborescences where k is the global min-cut
- Algo: Split-off a constant fraction of the vertices, recursively build arborescences for the rest, then put back the split-off vertices
- Putting back requires that the vertices split-off are independent
- By Turan's theorem, there is a n/k sized independent set
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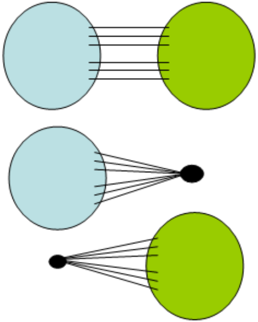
- Find global min-cut
- Create two Steiner min-cut sub-problems
- Consider computation tree



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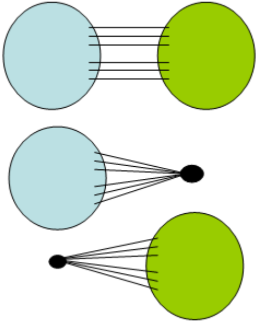

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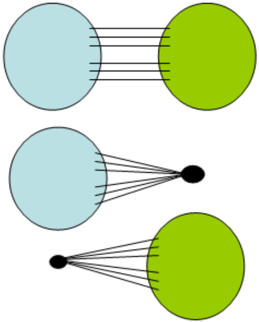

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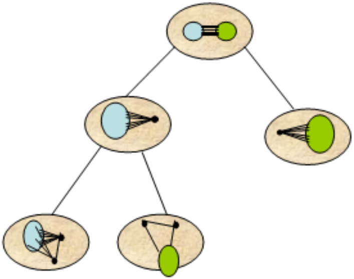

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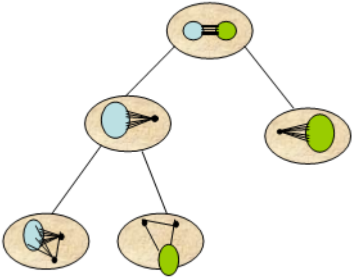

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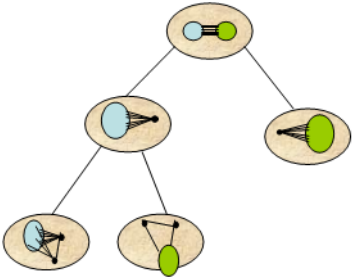

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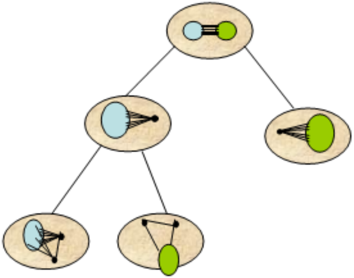

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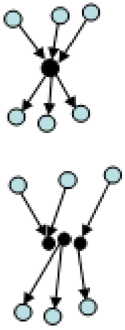

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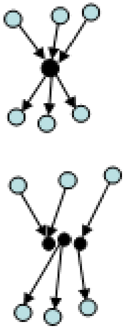

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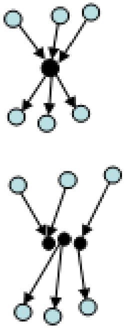

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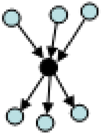

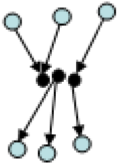

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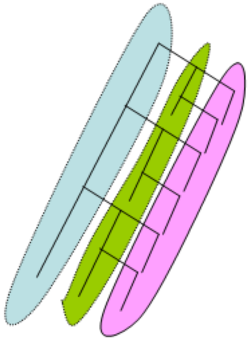

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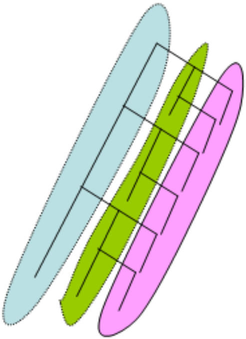

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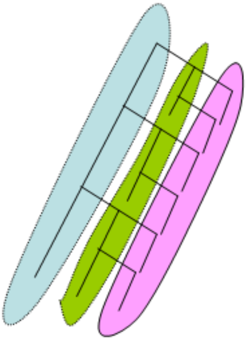

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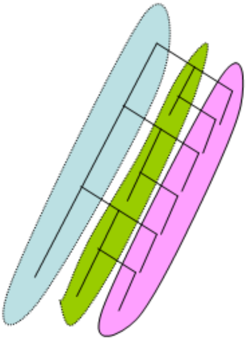

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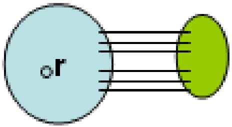

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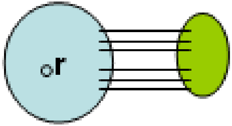

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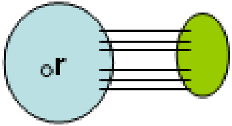

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- Given a graph with global min-cut 2k
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And Finally....

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